Journal of Risk and Financial Studies
Vol. 3, No. 1, 2022, pp. 25-53
https://doi.org/10.47509 /JRFS.2022.v03i01.02

ARF INDIA
Academic Open Access Publishing
www.arfjournals.com

Microfinance and Gender Discrimination in Credit Allocation: An Economic Analysis of Incentive Contracts

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Received: 22 January 2021; Revised: 23 February 2022;
Accepted: 5 March 2021; Publication: 30 June 2022

Abstract: *In this paper, we develop a model of discrimination by effort between women and men in credit markets. The specificity of our model compared to the current literature consists in the level of effort effected by the agents who benefited of a microcredit is endogenous. We assume that borrowers have financial constraints and that they want to obtain funds to carry out their projects whose success depends on costly and unobservable efforts. Assuming that women exert a higher level of effort than men, we show that when information is perfect, all projects are financed and the collateral is completely eliminated. Women benefit of a lower interest rate than men and at the same time receives a loan of the same amount as men. In a situation of imperfect information, when the level of the project manager's effort is private information, we show that in a competitive credit market where risks are not mixed, the riskier borrower obtains the same contract as in a perfect situation. When the state can guarantee high-risk borrowers (low efforts), their welfare improves. On the other hand, when government can provide guarantees for low-risk borrowers (high efforts), it reduces collective welfare.*

JEL: *D41 ; G21*

Keywords: *Microcredit; Gender; Discrimination; Incentive contracts; Information asymmetry; Loan guarantee.*

1. INTRODUCTION

Created to fight poverty in developing countries, microcredit has become a worldwide practice[2]. Over the past few years, microfinance has grown by more than 30 percent per year worldwide (in Europe, Russia, Africa, Latin America, India and Asia) and now affects more than 100 million people.

To cite this Article

Zaja Ratsimalahelo, Mathurin Founanou and Didier Hounwanou (2022). Microfinance and Gender Discrimination in Credit Allocation: An Economic Analysis of Incentive Contracts. *Journal of Risk and Financial Studies*, Vol. 3, No. 1, pp. 25-53. *https://doi.org/10.47509 / JRFS.2022.v03i01.02*

The majority of microcredit recipients are women. According to the Microfinance 2015 Barometer, women accounted for 81 percent of clients[3] of Microfinance Institutions (MFIs) in 2014. This evidence is shared in the economic literature[4]. While micro-credit recipients are predominantly women, few studies have been devoted to the conditions for the allocation of micro-loans. The most significant studies have revealed legal, social, cultural and economic restrictions that limit women's access to credit compared to men (Almeyda (1996), Lycette and White (1989), Sisto 1996)).

Agier and Szafarz (2013a) studied credit conditions, focusing on the phenomenon of discrimination through which women benefit of less credit than men for equivalent interest rates. In their studies, they show a psychological cost related to the financing of women who would devote more time to family life to the detriment of the project. This would increase the risk of default and justify the rationing practised by MFIs. This situation limits the opportunities for women to invest in large projects that can generate significant income.

Other studies have shown credit rationing towards women, they get lower loans than men (Brana (2012), Fletschner (2009) Garikipati *et al.* (2016)). According to Agier and Szafarz (2013b), gender discrimination mainly affects women who wish to obtain larger loans. Such discrimination is due to gender stereotypes of women entrepreneurs (Cozarenco and Szafarz (2018), Gupta *et al.* (2009)).

It is widely known that poverty is intimately linked to discrimination (Labie *et al.* (2015)). The difficulties faced by poor people (especially women) in providing the collateral requested by MFIs do not allow them to access to larger loans. To remedy this situation, the solidarity guarantee was developed by Grameen Bank in the 1970s. Its principle is to make the members of the group personally responsible for group loans. If a member of the group fails to do so, the other members have to repay the amount of the credit they received. By analyzing the determinants of loan repayment rates, Zeller (1998) shows that groups with higher levels of social cohesion have a better repayment rate. Conversely, Sadoulet (1997) analyzed the costs of joint liability while Diagne (1997) or Rai and Sjöström (2000) analyzed the conditions for its implementation. The findings of these works show that group loans do not result in better reimbursement rates than individual loans (Armendariz and Morduch (2010)). Moreover, Sadoulet (1997) has shown that "social guarantees" induced by group loans are not a sufficient condition to guarantee high reimbursement rates, while Diagne (1997) has pointed out the excessive costs of Joint liability. Rai and Sjöström (2000) have shown that the circumstances in which joint liability is optimal

are scarce and unlikely to hold in practice, dominated by a contract that provides truthful information about borrowers. They also argue that most group credit schemes fail to implement contracts fully. Thus, the results of this work on the impact of the group loan on the repayment rate are controversial. Godquin (2004) finds that the age of the group, a proxy for social ties inside the group, showed a significant negative impact on the reimbursement, which raises the question of the necessity of specific incentives instruments for experienced borrowers. By comparing the different credit allocation mechanisms in Microfinance, Sinn (2013) finds that the best mechanism for allocating credit depends on the distribution of projects and the amount of credit contracted. She finds that the simultaneous group loan works better when the probability of default is low and the likelihood of being able to repay two loans is high. These criticisms limit the development of group loans so that MFIs prefer individual loans today. That is why we develop in this paper specially an individual loan model.

According to the economic literature, women repay better than men (Armendariz and Morduch (2000; 2010), D'espallier *et al.* (2011), World Bank (2007)). The rate of reimbursement of women is very high (95 to 98 percent, according to Hofmann and Marius-Gnanou (2003)). The World Bank (2007) report indicates the high level of reimbursement among women may be related to their undertaking "conservative" investments and low moral hazard. In assessing different methods to reduce default, Armendariz and Morduch (2005) consider the female target as a technique in its own right alongside group loans and other dynamic incentives. Our point of view is that, this level of reimbursement is related to the effort of women in carrying out projects. According to Sambe and Agbobli (1997), women are increasingly reputed to "compensate by their seriousness for the weaknesses of their economic condition". This reputation, which represents a moral guarantee, is increasingly favored by MFIs that prefer to finance women. Lapenu *et al.* (2009) report that the development of systems based on solidarity and trust by MFIs receives recognition from beneficiaries of loans that repay more regularly.

It should be noted that women in developing countries are poorer[5] than men and invest in small projects (Agier and Szafarz (2013a)). The results of their work show that the guarantor and the gender of entrepreneur are not related. The approval rate is similar for men and women. However, women receive lower credits[6].

Similarly, it should be mentioned that the interest rates charged are very high (the example of the Grameen Banque is typical in this respect),

close to usurers' rates (Peemans-Pouillet (2000))[7]. Faced with this situation, public authorities (in West Africa) intervene in the markets by capping credit rates. However, the government's policy of capping the credit rate is a source of inefficiency. Helms and Reille (2004) pointed out the withdrawal of MFI's from the market. The latter no longer practice transparency on the total cost of credit and reduce their activities in rural areas. This government intervention has a negative impact on the coverage rate of microfinance in these geographical areas where coverage is already low compared to the rest of the world. Thus, this method of intervention would be unsustainable in terms of regulation of the micro-credit market.

The conditions of credit offered to these populations with modest incomes are today at the center of the debate. The question is whether these conditions are really part of the main mission of microfinance to fight poverty; so this type of contracts can improve the allocation of microloans to women and thus their situation.

The purpose of this study is to present a model of discrimination by effort between women and men in a credit markets. Taking into account the reimbursement effort is the originality of this paper compared to the work of Agier and Szafarz (2013a). Indeed, high reimbursement rates among women largely reduce psychological costs. In our model, credit is rationed for both moral hazard and adverse selection problems. Following Tirole (2001) and Holmström and Tirole (1997), we consider borrowers with financial constraints who want to obtain funds to carry out projects whose success depends on costly and unobservable efforts.

We assume that the borrower is able to solve the problem of credit rationing by offering as collateral the final endowments resulting from the project. However, collateral on its own resources may be insufficient to guarantee access to the credit market. We analyze the effects of a total guarantee of loans by the government. The guarantee of loans by the Government is now a common practice. Gudger (1998) and Beck *et al.* (2008) studied loan guarantee programs in developing countries and developed countries around the world. However, it is not clear, a priori, that guarantees necessarily improve social welfare. Guarantees only improve welfare when the private credit market is bankrupt, and government intervention in the credit market must introduce few distortions that it does not correct.

In this paper, we analyze the structure of loan contracts (interest rates, loan amounts and collateral) in a competitive credit market with asymmetric information when borrowers are risk neutral, and highlight the conditions under which government intervention can improve social welfare. Unlike classical literature on this subject (Bester (1987), Hellwig (1988), Besanko

and Thakor (1987)), our study includes repayment efforts by borrowers to analyze optimal contracts in asymmetric information on the types of credit applicants. On the base of the work by Schmidt-Mohr (1997), we characterize optimal contracts. We enrich the model by analyzing the impact of government intervention on welfare in credit markets by loan guarantees.

We found that in perfect information on the types of borrowers, all the projects are financed and the collateral is completely eliminated. Moreover, the marginal return on investment is equal to marginal social cost. In this situation, the MFI offers separate contracts to men and women and the comparison shows that women benefit from a lower interest rate than men and at the same time receives a loan of the same amount as men.

To characterize optimal contracts in imperfect information situations, we consider two situations: firstly, we assume that the collateral costs are lower than the final endowments of the borrowers and in the second, we assume that the final endowments of the borrowers are insufficient to cover collateral costs. We found that in a competitive credit market when risks are not mixed, the most risky borrower gets the same contract as in a perfect situation. We show that the effects of state intervention depend on how the incentive constraints, and in particular the collateral required, are affected. The loan guarantee of high-risk borrowers (low efforts) improves their well-being. On the other hand, the loans of low-risk borrowers (high efforts) reduce collective well-being.

To highlight our results, the rest of this article is organized as follows. In Section 2, we present a model of credit discrimination by effort between women and men, and we characterize optimal contracts for perfect information. In Section 3, we characterize optimal contracts for incomplete information. In section 4, we analyze the effects of a total guarantee of loans by the State. The conclusions and recommendations of economic policies are given in section 5.

2. THE BASIC MODEL

2.1. Description of the model

A credit market with two types of agent i (i=m for men and i=w women) and microfinance institutions (MFIs) is considered. All agents are assume neutral with respect to risk, and each agent has an investment project that is financed solely by borrowing. An agent of type i ($i = m, w$) can invest an I amount in a project that generates a random income $\tilde{Y}_i(I)$. The probability for the project to succeed depends on the effort e_i of the agent, which is not

observable by the MFI. If the agent exerts an effort $e_i \in [0,1]$ then the probability of success of the project equals to e_i.

In this context, the agent bears a cost of effort $\varphi(e_i)$ corresponding to the effort e_i where $\varphi(e_i)$ is strictly increasing, twice continuously differentiable and convex function, that is $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) > 0$ for $i = m, w$. We normalise $\varphi(0) = 0$.

As highlighted in the introduction, many studies have shown that women provide more effort than men. This allows us to assume that the cost of men's effort is lower than that of women, either $\varphi(e_m) < \varphi(e_w)$. In this case, the marginal cost of men's effort, which is none other than its probability of effort, is lower than that of women[8].

Thus, we can establish the following inequality according to which the probability of the women's effort is higher than men's:

$$0 < e_m < e_w < 1 \tag{1}$$

The distribution of project return is given by:

$$\tilde{Y}_l = \begin{cases} Y_i(I) & with\ probability\ e_i \\ 0 & with\ probability\ 1 - e_i \end{cases} \qquad i = m, w \tag{2}$$

Project returns $Y(I)$ are increasing and concave in I, such that $Y_i'(I) \succ 0$ and $Y_i''(I) \prec 0$ for $i = m, w$. We also assume that the optimal investment is bounded and is strictly positive with $Y_i(0) = 0$, $\lim_{I \to 0} Y_i'(I) = \infty$, and $\lim_{I \to \infty} Y_i'(I) = 0$ for $i = m, w$. $i = m,\ w$. (3)

Borrowers are risk-neutral and maximize expected returns. It is well known that a risky project generates a higher return on success than a less risky project. Thus, we assume that for any given volume of investment, the expected returns of investment projects for women and men are the same:

$$e_w Y_w(I) = e_m Y_m(I), \qquad \forall I \in [0, \infty[\tag{4}$$

According to condition (1), this equality of the expected returns of equation (4) means that men's projects are more risky than women's projects. This relationship can be apprehended by the fact that the effort made by women compensates exactly the least risk of their projects compared to men's projects. This condition is similar to that of Rothschild and Stiglitz (1976).

We assume that each agent has a wealth at the end of the project whose monetary value is denoted by $W > 0$.

Contracts. A contract $V_i(I_i, r_i, c_i)$ for $i = m, w$ specifies the amount of loan I_i, the interest rate charged on a loan r_i, with $r_i > r_0$ where $r_0 \neq 0$ is the cost of capital and $c_i \geq 0$ the amount of collateral required by MFIs.

Given a contract $V_i(I_i, r_i, c_i)$, the expected utility of an agent of type i is :

$$U_i(I_i, r_i, c_i, e_i) = e_i\left[Y_i(I_i) - (1+r_i)I_i + W\right] + (1-e_i)(W - c_i) - \varphi(e_i) \;\; i = m,w \tag{5}$$

The first term of equation (5) represents the expected income of entrepreneurs for $i = m, w$ when his project succeeds, the second term represents his expected income when the project fails and the third term represents the cost of the effort.

The agent selects his level of effort which must satisfy the following first-order condition:

$$Y_i(I_i) - (1+r_i)I_i + c_i = \varphi'(e_i) \tag{6}$$

Following equation (6) the marginal utility of the agent is equal to the marginal cost of the effort. This result is consistent with that obtained by Ghatak and Guinnane (1999) who have shown that the optimal level of effort is a decreasing function of the interest rate. Moreover, this result also shows that the level of effort is an increasing function of the collateral.

The MFI is assume to be risk-neutral, thus the expected profit of an MFI offering a credit contract $V_i(I, r, c)$ to agent i is given by:

$$E\pi_i(I, r, c) = e_i(1+r_i)I_i + (1-e_i)c_i - I_i(1+r_0) \;\text{ for } i = m, w. \tag{7}$$

Then for an optimal level of effort, which can be, obtain following equation (6), the constraint of zero profit (ZPC): $E\pi_i(I, r, c) = 0$ can be written by:

$$e_i(1+r_i)I_i + (1-e_i)c_i = I_i(1+r_0). \tag{8}$$

Let γ be the proportion of women in the population of borrowers. The proportion γ is independent of W. Given that the women receive the contract $V_w(I_w, r_w, c_w)$ and the men receive the contract $V_m(I_m, r_m, c_m)$, the IMF's expected profits is given by :

$$E[G(V_w, V_m)] = \gamma E\pi_w(I_w, r_w, c_w) + (1-\gamma)\, E\pi_m(I_m, r_m, c_m) \tag{9}$$

In the rest of this paper, we will assume that loanable funds are not scarce, i.e. MFIs can finance all investment projects regardless of the amount of the loan.

We will analyze preferences of borrowers and IMF over loan contracts by computing marginal rates of substitution (MRS) between contract parameters.

(a) Firstly, the corresponding MRS of borrowers can be computed from equation (5), the MRS of type i (henceforth MRS_i^b) between interest rate r and loan size I is given by

$$MRS_i^b(I/r) = \frac{Y'_i(I_i) - (1+r_i)}{I_i}, \text{ for } i = m, w \tag{10}$$

Note that $e_w > e_m$ and equation (4) imply $Y'_m(I) \succ Y'_w(I)$, for any $I > 0$. Thus, $MRS_m^b(I/r) \succ MRS_w^b(I/r)$ this result means if MFI's are willing to increase the size of a loan then men are more eager than women to pay a higher rate of interest.

The MRS of type i between collateral c and interest rate r is given by

$$MRS_i^b(r/c) = \frac{e_i}{1-e_i} I_i, \text{ for } i = m, w \tag{11}$$

Equation (11) implies $MRS_w^b(r/c) \succ MRS_m^b(r/c)$ for any positive volume of investment and any required collateral such as $0 \le c \le W$. This means that a contractor with a risky project in this case men are less willing than women to accept an increase in the collateral if the interest rate has dropped.

(b) Secondly, using equation (7) to be computed the MRS of MFI (henceforth, MRS_i^{mfi}) we have

$$MRS_i^{mfi}(r/I) = \frac{(1+r_0)/e_i - (1+r_i)}{I_i} \text{ for } i = m, w$$

and

$$MRS_i^{mfi}(c/r) = \frac{e_i}{1-e_i} I_i \text{ for } i = m, w \tag{12}$$

Comparing MRS of borrowers and of MFI, one obtains $MRS_i^b(r/I) = MRS_i^{mfi}(r/I)$ if and only if $Y_i^{'}(I) = (1+r_0)/e_i$, that is the expected marginal return on investment equals marginal cost of investment. Next, $MRS_i^b(c/r) = MRS_i^{mfi}(c/r)$ borrowers' and MFIs' MRS between collateral and interest rate coincide for any $I > 0$.

We will present the credit market equilibrium with perfect information. In this situation, the types of agents are identifiable ex-ante then MFIs face two distinct credit markets.

2.2. Credit contracts with perfect information

In this section, we consider that MFIs can observe both the type of agent and the nature of his project. This simplified analysis framework will characterize optimal contracts when agents choose the level of repayment effort corresponding to their type to be "high" for women and "low" for men.

A credit market equilibrium with perfect information is a menu of contracts (V_w, V_m) such that

(i) The IMF's expected profits is non negative,

(ii) There is no credit contracts (V_a, V_b) that give positive expected profits to a MFI offering a contract (V_a, V_b) in addition to (V_w, V_m).

These results refer to the notion of Nash equilibrium (see Bester 1987). Moreover, a credit contract offered in equilibrium must be feasible and maximize expected utility of type of agent subject to zero profit.

Thus, an optimal credit contract in equilibrium $V_i^*(I_i^*, r_i^*, c_i^*)$ for $i = m, w$ can be obtained to solve the following problem:

$$\max U_i(I_i, r_i, c_i)$$

s.t.

$$E\pi_i(I_i, r_i, c_i) = 0$$

$$(1+r_i)I_i \geq c_i \geq 0$$

$$Y_i(I_i) - (1+r_i)I_i + c_i - \varphi'(e_i) = 0$$

The solution of this program is given by the following proposition:

Proposition 1: *Credit market equilibrium with perfect information is characterized by the following contracts:*

(i) For the men: $V_m^*(I_m^*, r_m^*, c_m^*)$

$$c_m^* = 0$$

$$r_m^* = \frac{1+r_0}{e_m} \tag{13a}$$

$$I_m^* = Y_m'^{-1}\left(\frac{1+r_0}{e_m}\right)$$

(ii) For the women: $V_w^*(I_w^*, r_w^*, c_w^*)$

$$c_w^* = 0$$

$$r_w^* = \frac{1+r_0}{e_w} \tag{13b}$$

$$I_w^* = Y_w'^{-1}\left(\frac{1+r_0}{e_w}\right)$$

(iii) Where the optimal level of effort e^* is given by:

$$\varphi'(e_i) = Y_i(I_i) - (1+r_i)I_i \text{ for } i = m, w \tag{14}$$

See Appendix for Proof

In a competitive credit market with perfect information, all bankable projects are financed, the collateral is no longer needed, and the marginal return from investment is equal to the marginal social cost. Thus, equilibrium is therefore efficient.

A comparison of contracts V_w^* and V_m^* reveals that women pay a lower interest rate than men, since, $e_w \succ e_m$ we have

$$r_m^* = \frac{1+r_0}{e_m} \succ \frac{1+r_0}{e_w} = r_w^*$$

The agents who exert the highest level of effort especially women benefit from a lower interest rate.

Moreover, interest rate is decreasing in optimal effort. Indeed, we have:

$$\frac{dr_i^*}{de_i} = -\frac{1+r_0}{e_i^2} < 0 \qquad for\ i = m, w$$

3. EQUILIBRIUM WITH IMPERFECT INFORMATION

In reality, it is unlikely that the MFI will have all the information on the actions taken by the borrowers who submit projects for their financing. In fact, when a borrower obtains a loan, the performance of his project depends partly on the effort exerted for the success of his project. We have seen previously that in perfect information, the borrower chooses her/his level of effort such that the marginal gain is equal to the marginal cost. As a result, women pay a lower interest rate according to optimal contracts. However, when the information is asymmetric, such a result is no longer satisfied. Since the level of effort of the agents is not observable by the MFIs, it is therefore private information. Moreover, in practice, MFIs do not observe the risk of their clients' projects, so they are private information for them and these risks are likely to be low when the effort to implement the project is high. Subsequently the gender is observable, and the asymmetry of information relates to the risk of the project. In practice, some women have risky projects and some men have a low risk projects. In this section, without a loss of generality, we will assume the contract V_w corresponding to the safe borrower (good-risk) and the contract V_m to the risky borrower (bad-risk).

MFIs can offer a set of loan contracts that generate self-selection for each type of borrower in the appropriate contract. According to the theory of contracts, in a situation of competition, MFIs operate with zero profit and the contracts should be incentive, that is to say, satisfy the following constraints incentive compatible (IC) under imperfect information:

$$U_w(V_w) \geq U_w(V_m) \qquad \text{(ICI)} \qquad (15)$$

$$U_m(V_m) \geq U_m(V_w) \qquad \text{(IC2)} \qquad (16)$$

An equation (15) state that the safe type weakly prefers taking a contract V_w than a contract V_m, meanwhile equation (16) is satisfied when the risky type prefers taking a contract V_m than a contract V_w.

Any menu of incentive contracts (V_w, V_m) is individually rational for the MFI and the borrower, respectively, if

$$\gamma E\pi_w(V_w) + (1-\gamma)\, E\pi_m(V_m) \geq 0, \tag{17}$$

and

$$U_i(V_i) \geq 0 \text{ for } i = m, w \tag{18}$$

A competitive credit market equilibrium with imperfect information is a menu of contracts (V_w, V_m) such feasibility constraints of $0 \leq c_i \leq W$, $I_i(1 + r_i) \geq c_i$ and $I_i \geq 0$ hold for $i \in \{w, m)$, and individual rationality for MFIs and borrowers is satisfied. At this equilibrium, there is no menu of contracts (V_a, V_b), which can generate positive profits to MFIs offering a contract (V_a, V_b) in addition to the contract (V_w, V_m)

Note here that the contract of proposition 1 is not an incentive, since $r_w < r_m$ implies that both women and men prefer contract V_w^* to contract V_m^*. Thus, contract (V_w^*, V_m^*) cannot be an equilibrium contract in imperfect information. In the following, we will determine the conditions that guarantee the existence of a competitive equilibrium under imperfect information.

Any contract having the property $c_i = W$ violates the constraint of individual rationality. The immediate consequence is that the condition $c_i < W$ for $i \leq \{w, m\}$ must be checked at equilibrium. The condition $Y_w'(0) = Y_m'(0) = \infty$ ensures that a menu of incentive contracts (V_w, V_m) with a positive investment exists such that (V_w, V_m) is individually rational for borrowers and gives positive expected profits to the MFI that offers it. The problem we are considering here is therefore not trivial. Optimal contracts for imperfect information can therefore be determined using the traditional arguments of credit market literature (Besanko and Thakor (1987), and Bester (1987)).

In asymmetric information situation, the collateral is used as a separating mechanism (Besanko and Thakor (1987), Bester (1987), Chan and Thakor (1987), Stglitz and Weiss (1981)). High risk borrowers have a strong preference not to pay collateral, because they are most likely to face the obligation of repaying. The role of the collateral as a complete separation mechanism crucially depends on *W*, the level of the endowment of the borrower. In this section, we examine the market equilibrium when the endowment of the borrowers is sufficient to allow a complete separation. In section 4, we will examine the market equilibrium with an insufficient level of collateral and state intervention.

An equilibrium credit contract, $V_i^{**}\{I_i^{**}, r_i^{**}, c_i^{**}\}$ for $i \leq \{w, m\}$ is solution of the following program:

$$\underbrace{Max}_{I_i, r_i, c_i, e_i} \quad \gamma[U_w(I_w, r_w, c_w, e_w)] + (1-\gamma)[(U_m(I_m, r_m, c_m, e_m)]$$

s. t.

$$\begin{aligned} &\left\{e_w\left[Y_w(I_w) - (1+r_w)I_w + W\right] + (1-e_w)(W - c_w) - \varphi(e_w)\right\} \\ &\geq \left\{e_w\left[Y_w(I_w) - (1+r_m)I_m + W\right] + (1-e_w)(W - c_m) - \varphi(e_w)\right\} \end{aligned} \tag{19a}$$

$$\begin{aligned} &\left\{e_m\left[Y_m(I_m) - (1+r_m)I_m + W\right] + (1-e_m)(W - c_m) - \varphi(e_m)\right\} \\ &\geq \left\{e_m\left[Y_m(I_m) - (1+r_w)I_w + W\right] + (1-e_m)(W - c_w) - \varphi(e_m)\right\} \end{aligned} \tag{19b}$$

$$0 \leq e_i \leq 1, \; i \in \{m, w\} \tag{19c}$$

$$0 \leq c_i \leq (1+r_i)I_i, \; i \in \{m, w\} \tag{19d}$$

$$e_i(1+r_i)I_i + (1-e_i)c_i = (1+r_0)I_i \;\; i \in \{m, w\} \tag{19e}$$

$$Y_i(I_i) - (1+r_i)I_i + c_i - \varphi'(e_i) = 0 \;\; i \in \{m, w\} \tag{19f}$$

To determine the solution of this problem, we will consider two situations: the first situation consists in supposing that the collateral costs are lower than the final endowments of the borrowers $c_i^* < W$ and the second situation supposes that the final endowments of the borrowers are sufficient to cover the collateral costs $c_i^* < W$.

In the first case, when the collateral is unconstrained, we obtain the following proposition.

Proposition 2: *When information is imperfect on the competitive credit market, optimal contracts are characterized by the following conditions:*

(i) For the risky borrowers: $V_m^{**}(I_m^{**}, r_m^{**}, c_m^{**})$

$c_m^{**} = 0$

$$r_m^{**} = \frac{1+r_0}{e_m} - 1$$

$$I_m^{**} = Y_m^{'-1}\left[\frac{\left(1-\gamma+\lambda_1\frac{1}{e_m}-\lambda_3\frac{1}{e_m}+\lambda_5\frac{1}{e_m}\right)(1+r_0)}{(1-\gamma)e_m+\lambda_5}\right]$$

(ii) For the safe borrowers: $V_w^{**}(I_w^{**}, r_w^{**}, c_w^{**})$

$$c_w^{**} = \frac{(1+r_0)(e_w I_m - e_m I_w)}{e_w - e_m}$$

$$r_w^{**} = \frac{1+r_0}{e_w} - \frac{(1-e_w)c_w^{**}}{e_w I_w} - 1$$

$$I_w^{**} = Y_w^{'-1}\left[\frac{\left(\gamma+\lambda_1\frac{e_m}{e_w}-\lambda_2\frac{1}{e_w}+\lambda_4\frac{1}{e_w}\right)(1+r_0)}{\gamma e_w+\lambda_4}\right]$$

(iii) The levels of effort are given by the following condition:

$$\varphi'(e_i) = Y_i(I_i^{**}) - (1+r_i^{**})I_i^{**} + c_i^{**} \text{ for } i = m, w$$

See Appendix for Proof

In a competitive credit market, when the risks are not mixed, the zero profit condition must be checked: $\pi_w(V_w) = \pi_m(V_m) = 0$.

In this situation where the type of risk of each borrower's project is private information, MFIs can propose a set of contracts that induce borrowers to select the appropriate contracts (self-selection). At equilibrium, the result of Proposition 2 shows that high-risk borrowers (men) choose a contract with a relatively high interest rate and the MFI does not require collateral. They thus obtain the same contract as in situation of perfect information, that is to say

$V_m^{**} = V_m^*$ and e^{**} is solution of the equation $\varphi'(e_m) = Y_m(I_m^*) - (1 + r_m^*)$. The riskier borrower does not have collateral and accepts to pay a higher rate of interest: $r_m^{**} \succ r_w^{**}$. In contrast, collateral is required for less risky borrowers. Their optimal level of effort is a decreasing function of the interest rate and an increasing function of the collateral (Ghatak and Guinnane (1999)). Their projects are therefore not always funded if they do not have collateral.

This result shows a kind of discrimination between men and women to the extent that a guarantee is demanded from women who agree to provide more effort for lower rates. On the other hand, men who give less effort are asked to pay higher interest rates and zero collateral.

The existence of a separating equilibrium can be ensured by assuming that there are a sufficiently large proportion of high-risk borrowers, or that the difference between them $e_w - e_m$ is large enough[9].

As long as the low risk (women) will have enough resources to guarantee the loan (in terms of collateral), there will be no rationing at equilibrium. Nevertheless, because of the loss of efficiency caused by the use of collateral, there is a potential role for the state (Gale, 1989).

The main result here is that credit policies are characterized by their effects on the incentive constraint that determine the set of possible contracts such as high-risk borrowers cannot take low-risk contracts. Thus, the policy of securing loans for low-risk (women) borrowers reduces their interest rates. Since high-risk credit agreements do not change, i.e. a high interest rate and no collateral, in order to restore the incentive constraint, the collateral required for low risk must be increased. Therefore, guarantees for low risk reduce efficiency.

However, when low risk can announce a small amount of collateral, low risk contracts should be made less desirable in order to restore the incentive constraint. The only option is to reduce the probability of securing a low risk loan, i.e., introducing a rationing of low risk borrowers. With the existence of rationing, it is plausible to envisage credit policies with state intervention.

The inability to provide the required collateral for less risky borrowers under imperfect information is an obstacle to the financing of their projects. Thus, to make up for this, it may be better for the state to intervene to guarantee the projects.

4. EQUILIBRIUM OF THE CREDIT MARKET WITH STATE INTERVENTION

In this section, we focus on state intervention in the microfinance sector. This intervention doesn't only improve social welfare, but also helps foster

economic development. Several types of intervention are possible: investment financing, loan guarantees, interest rate subsidies and various subsidies. Our analysis focuses on the guarantee of loans by the state.

4.1. Equilibrium in the presence of a loan guarantee by the State

The loan guarantee ensures that MFIs receive an amount ρ_i, with $0 \leq \rho_i < I_i$. The government can fix $\rho_w = \rho_m = \rho$, or choose the ρ_i individually. In return for the guarantee, the MFIs pay all the collateral collected to the government. The government's net cost of a default on a loan is therefore defined by $\rho_i - c_i$, where c_i represents the collateral required in the presence of guarantee. It should be noted here that the government is subject to the same informational constraints as MFIs face borrowers.

With the government loan guarantee, the expected utility of borrowers is always given by equation (5). The zero profit condition for MFIs is now given by:

$$e_i (1 + r_i) I_i + (1 - e_i) \rho_i = (1 + r_0) I_i, \text{ for } i \in \{w, m\} \tag{20}$$

The solution of the problem is therefore to replace the equation (19e) by the equation (20) and the solution of this problem is given by the following proposition:

Proposition 3: *When the level of effort of the borrower is a private information and if* $W > c_w^*$, *a competitive market equilibrium with loan guarantee by the state is characterized by the following conditions:*

(i) For the risky borrowers: $V_m^{***}(I_m^{***}, r_m^{***}, c_m^{***})$

$$c_m^{***} = 0$$

$$r_m^{***} = \frac{1+r_0}{e_m} - 1$$

$$I_m^{***} = Y_m^{'-1}\left[\frac{\left((1-\gamma) + \lambda_1 \frac{1}{e_m} - \lambda_3 \frac{1}{e_m} + \lambda_5 \frac{1}{e_m}\right)(1+r_0)}{(1-\gamma)e_m + \lambda_5}\right]$$

(ii) For the safe borrowers: $V_w^{***}(I_w^{***}, r_w^{***}, c_w^{***})$

$$c_w^{***} = \frac{(1+r_0)(e_m I_w - e_w I_m)}{e_m(1-e_w)\rho_w - (1-e_m)}$$

$$r_w^{***} = \frac{1+r_0}{e_w} - \frac{(1-e_w)}{e_w I_w}\rho_w - 1$$

$$I_w^{***} = Y_w^{'-1}\left[\frac{\left(\gamma + \lambda_1 \frac{e_m}{e_w} - \lambda_2 \frac{1}{e_w} + \lambda_4 \frac{1}{e_w}\right)(1+r_0)}{\gamma e_w + \lambda_4}\right]$$

(iii) The levels of effort are given by the following condition:

$$\varphi'(e_i) = Y_i(I_i^{***}) - (1+r_i^{***})I_i^{***} + c_i^{***} \text{ for } i = m, w$$

See Appendix for Proof

In the proposition 2, without loan guarantee by government, the MFI receives respectively c_w^{**} and 0 as collateral on the loans of type borrowers w and m. We can verify that if $\rho_w = c_w^*$ and ρ_m = 0, the equilibrium of proposition 3 is reduced to the equilibrium of proposition 2. Only high guarantee rates produce an effect. The results of proposition 3 show that when guarantee increases, the collateral decreases. Proposition 3 gives the conditions under which state intervention can improve the efficiency of the credit allocation policy, although this intervention involves a social cost. These results are consistent with Gale (1990) who has dealt with a question similar to that posed here: when the state intervenes, can we improve social welfare? In the framework of the model developed here, a question similar to that posed by Gale (1989) can be analyzed. Our aim is to seek the role of guaranties in the effective financing of women's projects (in terms of reduction of discrimination) in a context where men and women are distinguished by the level of effort and the type of risk of the project. If our model has conceptual similarities with Gale (1990), it is clear that our model differs in taking into account both adverse selection and moral hazard.

Gale (1989) analyzed the effects of state intervention in the credit market when banks use collateral, interest rates and the probability of securing a loan as potential selection mechanisms. Recall here that state intervention in credit markets is a common practice, and many studies on the impact of this intervention lead to mixed results (Founanou and Ratsimalahelo (2010), Gale (1989, 1990)). We seek here to specify under what conditions this type of intervention is effective, admitting that MFIs practice a variable interest rate. In practice, MFIs tend to operate at fixed interest rates by type of credit

/ financial product. Unlike Gale (1989) and Founanou and Ratsimalahelo (2010), we take into account existing inequalities between men and women (especially in the effort and risk of the project) to analyze optimal contracts. It is a matter of determining the type of contract that can improve the conditions for granting the loan (here, we are interested in particular interest and collateral). An important result highlighted: the intervention of the state through the provision of guarantees promotes the access of low-risk borrowers (women) to credit.

In the absence of rationing, the government's natural policy is characterized by the probability μ_g to obtain a loan guarantee since one cannot obtain a private loan, and ρ_w the amount of the state guarantee. Note that for a risky borrower, $\rho_m = 0$ as $c_w^* = 0$, since he is willing to pay a higher interest rate for not paying the collateral, the state guarantee does not intervene. His situation is not influenced by state policy.

The interest rate on the secured loan, given the condition of zero profit is written:

$$r_g = \frac{1+r_0}{e_w} - \frac{(1-e_w)}{e_w I_w}\rho_w - 1 \tag{21}$$

To highlight the effects of credit policy on the well-being of borrowers, we consider a utility function composed of two parts, the first is provided by private financing and the second by public funding as it has done (see Gale (1990)). It is assumed that the government collects W as collateral. With such a policy, the utility of the safe borrower is given by:

$$U_w = \mu_w X_{ww} + (1-\mu_w)\mu_g X_{wg} \tag{22}$$

where $X_{ww} = X_{wg} = e_w\left[Y_w(I_w) - (1+r_w)I_w + W\right] + (1-e_w)(W-c_w) - \varphi(e_w)$

and $X_{wg} = e_w\left[Y_w(I_w) - (1+r_g)I_w + W\right] + (1-e_w)(W-c_w) - \varphi(e_w)$.

In equation (22) the first term represents the probability of obtaining a private loan μ_w multiplied by X_{ww} the expected utility of the low-risk borrower when he obtains an average private loan from a low-risk borrower. The second term represents the probability of obtaining a state-guaranteed loan, $(1-\mu_w)\mu_g$, multiplied by X_{wg} the expected utility of the low-risk borrower for that loan.

If $\rho_w = W$, then $X_{ww} = X_{wg} = e_w\left[Y_w(I_w) - (1+r_w)I_w + W\right] + (1-e_w)(W-c_w) - \varphi(e_w)$ and there is no gain from getting a government loan rather than a private loan.

If $\rho_w > W$, then $X_{ww} \succ X_{wg}$. The incentive constraint (17b) is then written:

$$\begin{aligned} &\mu_m \left\{ e_m \left[Y_m(I_m) - (1+r_m) I_m + W \right] + (1-e_m)(W - c_m) - \varphi(e_m) \right\} \\ &\geq \mu_w X_{mw} + \left(1-\mu_w\right) \mu_g X_{mg} \end{aligned} \tag{23}$$

where $X_{mw} = e_m \left[Y_m(I_m) - (1+r_w) I_w + W \right] + (1-e_m)(W - c_w) - \varphi(e_m)$ is the expected utility of the risky borrower who takes an average private loan for a safe borrower, and $X_{mg} = e_m \left[Y_m(I_m) - (1+r_g) I_w + W \right] + (1-e_m)(W - c_w) - \varphi(e_m)$ is the expected utility of the risky borrower who takes a medium loan from the government for a safe borrower. Therefore, we obtain the following proposition.

Proposition 4: *When the level of effort of the borrower is private information, and $W < c_w^{**}$, a credit market equilibrium with loan guarantee is characterized by the following conditions:*

(i) For the risky borrowers: $V_m^{***}(I_m^{***}, r_m^{***}, c_m^{***})$

$$c_m^{***} = 0$$

$$\mu_m^{***} = 1$$

$$r_m^{***} = \frac{1+r_0}{e_m} - 1$$

$$I_m^{***} = Y_m^{'-1} \left[\frac{(1+r_0)\left((1-\gamma) + \lambda_1 \mu_m - \lambda_3 \frac{1}{e_m} + \lambda_5 \frac{1}{e_m} \right)}{(1-\gamma) e_m + \lambda_1 \mu_m e_m - \lambda_1 \mu_w e_m - \lambda_1 (1-\mu_w) \mu_g + \lambda_5} \right]$$

(ii) For the safe borrowers: $V_w^{***}(I_w^{***}, r_w^{***}, c_w^{***})$

$$c_w^{***} = W$$

$$\mu_w^{***} = \frac{e_m \left(Y_m(I_m) - (1+r_m) I_m + W \right) - \mu_g X_{mg} - \varphi(e_m)}{(1-\mu_g) X_{mg}} \prec 1$$

$$r_w^{***} = \frac{1+r_0}{e_w} - \frac{(1-e_w)}{e_w I_w} W - 1$$

$$I_w^{***} = Y_m^{'-1}\left[\frac{(1+r_0)\left(\gamma-\lambda_1\mu_w\frac{e_m}{e_w}-\lambda_1(1-\mu_w)\mu_g\frac{e_m}{e_w}-\lambda_2\frac{1}{e_w}+\lambda_4\frac{1}{e_w}\right)}{\gamma e_w+\lambda_4}\right]$$

See Appendix for Proof

Private loans are supplemented by state-guaranteed loans for some low-risk borrowers characterized by (ρ_g, r_g, W). The risky borrowers get the same contract and utility as in propositions 2 and 3.

When the state agrees to offer subsidized credit (direct or indirect (loan guarantee)) for a proportion of low-risk borrowers who are excluded from the private market (women), these contracts may become more attractive for high risk. It must therefore be made less desirable to restore the incentive constraint. Since the required collateral cannot increase, the only alternative is to reduce the probability of obtaining loans (public and private). That is, increasing subsidies for rationed borrowers increases the extent of rationing.

4.2. Effects of state loan guarantees on social welfare

In general, the use of collaterals leads to a loss of efficiency. The results of propositions 3 and 4 should have important implications in terms of social welfare.

According to proposition 3, an increase of state loan guarantee ρ_w leads to an increase of utility of the safe borrowers U_w, although it increases the collateral c_w. For all $\rho_w \succ c_w^*$ the safe borrowers have greater utility than in an equilibrium without state loan guarantees. An increase of state loan guarantee ρ_m leads to an increase of utility of both safe and risky borrowers U_w and U_m respectively. Therefore, the state loan guarantee improves the situation of both types of borrowers.

When the initial allocation is characterized by proposition 4, an increase in ρ_g or ρ_w reduces the probability for the low-risk borrower to obtain a loan and thus increase the extent of rationing. In fact, when the state subsidizes borrowers who cannot obtain private financing, it increases the number of borrowers who cannot obtain public or private financing. This result is based on the response to the equilibrium of MFIs, in relation to the incentive constraint.

It should be noted, however, that while subsidizing borrowers increases the probability of a low-risk borrower being rationed, government loan guarantees increase the ex-post utility of those receiving government credits, given the reduction in the interest rate r_g on secured loans.

The determination of social welfare is based on the sum of borrowers' utility expectations and the MFI's overall profit minus the cost of guarantee funds born by the government. Social welfare can be defined by:

$$V_c = \gamma U_w + (1-\gamma)U_m - \gamma(1-e_w)(\rho_w - c_w) - (1-\gamma)(1-e_m)(\rho_m - c_m) \qquad (24)$$

This expression analyzes the effects of government intervention on collective welfare.

The effects of state guarantees on welfare may be characterized by the following propositions.

Proposition 5: *When the initial allocation is given by the proposition 3, that is, the level of the borrower's effort is private information and if* $W \succ c_w^*$, *the welfare effects of the loan guarantee are characterized by the following results:*

$$\left.\frac{\partial V_c}{\partial \rho_w}\right|_{\bar{\rho}_m} = \begin{cases} 0 & if \quad \rho_w \prec c_w^* \\ \prec 0 & if \quad \rho_w \geq c_w^* \end{cases}$$

$$\left.\frac{\partial V_c}{\partial \rho_m}\right|_{\bar{\rho}_w} \succ 0$$

$$\left.\frac{\partial V_c}{\partial \rho}\right|_{\rho=\rho_w=\rho_m} \succ 0$$

The main result of the proposition (5) is that, the effects of government intervention depend on how the incentive constraint, and in particular the collateral required are affected.

The second result states that subsidizing borrowers in the high-risk group improves their welfare.

Proposition 6*: When the initial allocation is given by the proposition 4, the effects of government intervention on welfare are characterized by:*

$$\frac{\partial V_c}{\partial \rho_w} \prec 0$$

$$\frac{\partial V_c}{\partial \rho_g} \prec 0$$

These conditions show that, the guarantees of low-risk borrowers reduce social welfare. Indeed, an increase in rationing represents a loss of efficiency.

Although subsidies increase the extent of rationing, they increase ex-ante the expected utility of low-risk borrowers. Indeed, the benefit of an additional loan from the state reduces the cost of increasing a probability of rationing. For example, low-risk subsidies improve ex-ante the well-being of borrowers, but actually reduce the utility of some low risk ex-post borrowers. As they increase the extent of rationing, low risk subsidies reduce efficiency overall.

On the other hand, subsidizing high risks entails a loss of incentive. The consequence is that the extent of low risk rationing is decreasing and efficiency is increasing (Gale, 1989).

The results of the impact of state intervention on the micro credit market through loan guarantees have shown improved access to microcredit for poor people.

However, the work of Gale (1990) indicated that loan guarantees are more effective than any form of direct grant from project sponsors. Bourlès and Cozarenco (2014) will later show that this result is no longer necessarily true when the subsidy is indirect. They also find that government loan guarantees can have a counter-productive effect by reducing the number of loan recipients.

Several empirical studies corroborate these findings: Craig et al. (2007) highlighted a positive impact of loan guarantees on small business development in the US and local economic growth. In a subsequent study, Craig et al. (2008) will show that the guarantee of loans by the state improves the general level of employment in the local economy.

CONCLUSION

In this paper, we examined the optimal contracts in the competitive credit markets, taking into account the repayment effort. According to economic literature, reimbursement rates are very high (between 95 percent and 98 percent). However, credit rates are very high (close to usurers' rates) and men benefit from better conditions while women efforts to reimburse are higher. This discrimination is linked to insufficient physical guarantees provided by women, while in developing countries women's resources are modest.

By integrating effort, we find that in perfect information, collateral is completely eliminated and women get lower interest rates than men, while men and women benefit from the same credit size. In imperfect information, the collateral is used as a separating mechanism. We find that when the risks are not mixed, the riskier borrower gets the same contract as in perfect information. By studying government intervention in credit markets through loan guarantees, we find that this guarantee improves the accessibility of low-risk borrowers, while high-risk borrowers obtain the same contract and the same utility as in imperfect information. The impact of state intervention on collective welfare is positive but needs to be qualified. When the state can guarantee high-risk borrowers (low efforts), their welfare improves.

On the other hand, when government can provide guarantees for low-risk borrowers (high efforts), it reduces social welfare. Indeed, we can see perverse effects from women with modest incomes who would make less and less efforts with the guarantee of the government. Thus, women would turn to riskier projects. Consequently, allocative efficiency is no longer guaranteed with government intervention.

6. APPENDIX

Proof of Proposition 1

In each sub-market, the equilibrium contract maximizes the utility's expectation of utility, U_i under the constraint of zero profit.

$$\begin{cases} Max U_i(I_i, r_i, c_i) \\ s.t \\ e_i(1+r_i)I_i + (1-e_i)c_i = (1+r_0)I_i \\ (1+r_i)I_i \geq C_i \geq 0 \\ Y_i(I_i) - (1+r_i)I_i + c_i - \varphi'(e_i) = 0 \end{cases}$$

By substituting the condition of zero profit in the utility function of each agent that is $(1+r_i)I_i = \frac{(1+r_0)I_i}{e_i} - \frac{(1-e_i)c_i}{e_i}$, we obtain the following program:

$$\underset{I_i, r_{Ii}, c_{Ii}}{Max}\, L^* = e_i\left\{Y_i(I_i) - \frac{I_i}{e_i}(1+r_0) + \frac{(1-e_i)}{e_i}c_i + W\right\} + (1-e_i)(W-c_i) - \varphi(e_i)$$

$$+ \lambda_1\left[\frac{I_i}{e_i}(1+r_0) - \frac{1-e_i}{e_i}c_i - c_i\right] + \lambda_2\left[Y_i(I_i) - \frac{I_i}{e_i}(1+r_0) + \frac{1-e_i}{e_i}c_i + c_i - \varphi'(e)\right]$$

The first order conditions give the following results:

$$\frac{\partial L^*}{\partial c_i} = -\lambda_1 \frac{1}{e_i} + \lambda_2 \frac{1}{e_i} = \frac{1}{e_i}(-\lambda_1 + \lambda_2) \leq 0$$

This implies that $c^*_i = 0$, and according to the zero profit condition, we have:

$$r_i^* = \frac{1+r_0}{e_i} - 1$$

This relationship shows that borrowers who exercise a higher level of effort, particularly women $(e_m \prec e_w)$, pay a lower interest rate than men.

$$r_m^* = \frac{1+r_0}{e_m} - 1 \succ \frac{1+r_0}{e_w} - 1 = r_w^*$$

Moreover, we have:

$$\frac{dr_i^*}{de_i} = -\frac{1+r_0}{e_i^2} \prec 0$$

We have also

$$\left.\frac{\partial L}{\partial I_i}\right|_{c_i^*=0} = e_i\left[Y_i^{'}(I_i) - \frac{1+r_0}{e_i}\right] + \lambda_1\left(\frac{1+r_0}{e_i}\right) + \lambda_2\left[Y_i^{'}(I_i) - \frac{1+r_0}{e_i}\right] = 0$$

By fixing $\lambda_1 = 0$ and $\lambda_2 > 0$, we have :

$Y_i'(I_i) - \frac{1+r_0}{e_i} = 0$, thus we have $I_i' = Y_i'^{-1}(\frac{1+r_0}{e_i})$.

Proofs of Propositions 2, 3 and 4

The determination of equilibrium conditions follows a common approach. The results of Proposition 2 are given in detail, while the results of the following proposals will be shortened.

When the information is asymmetric, the competitive market equilibrium is obtained by maximizing the weighted average borrowers' utility expectations, given by:

$$\gamma U_w + (1-\gamma)U_m \tag{A-1}$$

under the constraints (17a) - (17f). Following the approach of Besanko and Thakor (1987), the technique consists in first ignoring the constraints (17a) and then verifying that the optimal solution satisfies (17a).

By substituting $(1 + r_i)I_i$ by its value from the zero profit constraint ($E\pi = 0$), $\dfrac{(1+r_0)I_i - (1-e_i)c_i}{e_i}$ in the objective function, we obtain the following problem:

$$\begin{aligned}
&\max_{I_w, r_w, e_w} L = \gamma\left\{e_w\left[Y_w(I_w) - \frac{I_w}{e_w}(1+r_0) + \frac{1-e_w}{e_w}c_w + W\right] + (1-e_w)(W-c_w) - \varphi(e_w)\right\} \\
&(1-\gamma)\left\{e_m\left[Y_m(I_m) - \frac{I_m}{e_m}(1+r_0) + \frac{1-e_m}{e_m}c_m + W\right] + (1-e_m)(W-c_m) - \varphi(e_m)\right\} \\
&+\lambda_1\left\{e_m\left[Y_m(I_m) - \frac{I_m}{e_m}(1+r_0) + \frac{1-e_m}{e_m}c_m + W\right] + (1-e_m)(W-c_m) - \varphi(e_m) - e_m\left[Y_m(I_m) - \frac{I_w}{e_w}(1+r_0) + \frac{1-e_w}{e_w}c_w + W\right] - (1-e_m)(W-c_w) + \varphi(e_m)\right\} \\
&+\lambda_2\left[\frac{I_w}{e_w}(1+r_0) - \frac{1-e_w}{e_w}c_w - c_w\right] + \lambda_3\left[\frac{I_m}{e_m}(1+r_0) - \frac{1-e_m}{e_m}c_m - c_m\right] \\
&+\lambda_4\left[Y_w(I_w) - \frac{I_w}{e_w}(1+r_0) - \frac{1-e_w}{e_w}c_w + c_w - \varphi'(e_w)\right] + \lambda_5\left[Y_m(I_m) - \frac{I_m}{e_m}(1+r_0) - \frac{1-e_m}{e_m}c_m + c_m - \varphi'(e_m)\right]
\end{aligned} \tag{A-2}$$

where λ_1, λ_2, λ_3, λ_4, and λ_5 are are the Lagrange multipliers associated with the constraints of the program.

By differentiating the Lagrangian with respect to c_w, we obtain:

$$\frac{\partial L}{\partial c_w} = -\lambda_1\left(\frac{e_m - e_w}{e_w}\right) + \lambda_2\left(\frac{1-2e_w}{e_w}\right) + \lambda_4\left(\frac{1}{e_w}\right) \leq 0$$

By differentiating with respect to c_m, we obtain:

$$\frac{\partial L}{\partial c_m} = (-\lambda_3 + \lambda_5)\left(\frac{1}{e_m}\right) \prec 0$$

Therefore, $c_m^{**} = 0$. Substituting c_m^{**} by its value in (17b) and solving (17b) with respect to c_m^{**} we obtain:

$$c_w^{**} = \frac{\left[e_w I_m - e_m I_w\right](1+r_0)}{e_w - e_m}$$

Given c_i^{**}, for $i \in \{m, w\}$, the value of r_i^{**} can be obtained using the condition (17e). By deriving the Lagrangian with respect to I_i, we obtain the values of I_w^{**} and I_m^{**}.

Thus for safe borrower, we have

$$\frac{\partial L}{\partial I_w} = e_w \left[Y_w^{'}(I_w) - \frac{1+r_0}{e_w} \right] + \lambda_1 e_m \left[\frac{1+r_0}{e_w} \right] + \lambda_2 \left[\frac{1+r_0}{e_w} \right] + \lambda_4 \left[Y_w^{'}(I_w) - \frac{1+r_0}{e_w} \right] = 0 \qquad \text{A-3}$$

$$\frac{\partial L}{\partial c_w} = \lambda_1 e_m \left[\frac{1-e_w}{e_w} + (1-e_m) \right] + \lambda_2 \left[\frac{1-e_w}{e_w} - 1 \right] - \lambda_3 \frac{1-e_w}{e_w} \leq 0 \qquad \text{A-4}$$

With $c_w \succ 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$.

Notes

1. International organizations such as the United Nations (UN) and the United Nations Development Program (UNDP) place microfinance at the forefront of their poverty reduction goals. They finance 3% of MFI funds.
2. This rate reaches more than 80% of poor clients.
3. Armendàriz and Morduch (2000).
4. This rate reaches more than 80% of poor clients.
5. In Brazil, the average credit for women is BRL 846 and BRL 1074 for men.
6. Mauk and Diener (2012) found that the effective interest rate is 16.59% and not 20% if the default time of about 4 weeks is taken into account.
7. A simple way of illustrating this idea is to assume that the function of the cost of effort is of the quadratic type, it is therefore an increasing and convex function.

   That is : $\varphi(e) = \frac{1}{2}e^2$.
8. If $\varphi(e_m) < \varphi(e_w)$, then we have $\varphi'(e_m) = e_m < e_w = \varphi'(e_w)$. Cf. Rothschild and Stiglitz (1976) or Besanko and Thakor (1987). There is no mixing equilibrium under the assumptions of this section.

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